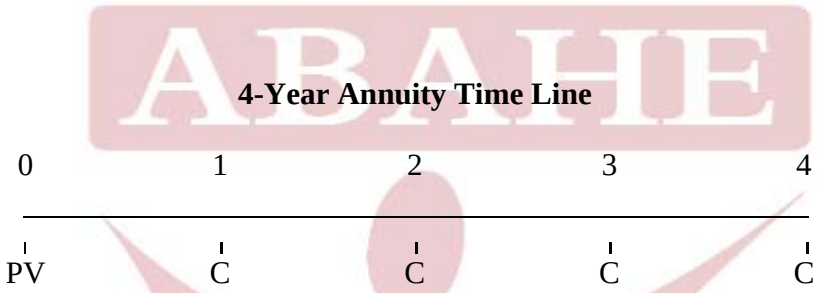
Annuities

An **annuity** is a series of equal payments over a specified time frame. For example, a cash payment of *C* made at the end of each year for four years at annual interest rate *i* is shown in the following time line:



This time line is for an *ordinary annuity*, in which the cash payments are made at the end of each year. For example, the first payment is made exactly one year from the present. The present value of this cash flow is calculated by:

$$PV = C/(1+i) + C/(1+i)^2 + C/(1+i)^3 + C/(1+i)^4$$

In general, for a *t* year annuity:

$$PV = C/(1+i) + C/(1+i)^2 + \ldots + C/(1+i)^t$$

From this potentially long series, a present value formula can be derived. First, multiply each side by $1/(1+i)$.

$$PV/(1+i) = C/(1+i)^2 + C/(1+i)^3 + \ldots + C/(1+i)^{t+1}$$

In order to eliminate most of the terms in the series, subtract the second equation from the first equation:

$$PV \; - \; PV/(1+i) \; = \; C/(1+i) \; - \; C/(1+i)^{t+1}$$

Solving for *PV*, the present value of an ordinary annuity is given by:

$$\boldsymbol{PV \; = \; \frac{C}{i}\left[1 \; - \; \frac{1}{(1+i)^{t}}\right]}$$

This equation assumes that the first payment of the annuity is made at the end of the first time period. If instead the payments are made at the beginning of each time period, then the present value calculation would be similar to the above, except that all payments would be shifted forward by one year. This shift can be accomplished by multiplying the entire present value expression by (1 + *i*). Such an annuity with the payments occurring at the beginning of each time period is called an *annuity due*.

Annuity Factor Table

The factor for calculating the present value of an ordinary annuity may be calculated for a range of time periods and interest rates and tabulated for quick reference. The annuity factor is the value of the following expression:

$$\frac{1}{i}\left[1 \; - \; \frac{1}{(1+i)^{t}}\right]$$

The following table shows the value of this factor for various interest rates and time periods.

Table of Present Value Annuity Factors

$_t$ \ ^{i}	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606