If the Economy is Strong

The expected value of the **large store**, given that the economy is strong (Decision Node 2, Probability Node 4), is:

$$EV\ (\text{Node } 4) = (.75 \times \$10{,}000{,}000) + (.25 \times \$4{,}000{,}000) = \$8{,}500{,}000$$

The expected value of the **small store**, given that the economy is strong (Decision Node 2, Probability Node 5), is:

$$EV\ (\text{Node } 5) = (.75 \times \$7{,}000{,}000) + (.25 \times \$5{,}000{,}000) = \$6{,}500{,}000$$

If the Economy is Weak

The expected value of the **large store**, given that the economy is weak (Decision Node 3, Probability Node 6), is:

$$EV\ (\text{Node } 6) = (.25 \times \$4{,}000{,}000) + [.75 \times \$(1{,}000{,}000)] = \$250{,}000$$

The expected value of the **small store**, given that the economy is weak (Decision Node 2, Probability Node 7), is:

$$EV\ (\text{Node } 7) = (.25 \times \$2{,}000{,}000) + (.75 \times \$500{,}000) = \$875{,}000$$

We have now reduced the decision tree from 8 branches to 4 branches. We next move back to decision Nodes 2 and 3 and select the alternative that leads to the best expected value at each.

Once the expected value of the various outcomes is calculated, based on the payoff amounts and the probability of the outcome, then at each preceding decision node (not probability node) we can choose the alternative that we calculated with the best expected value.

So at decision Node 2 (the economy is strong), we will select the large store because its expected value (EV = $8,500,000) is greater than the expected value of the small store (EV = $6,500,000). At decision Node 3 (the economy is weak), we will select the small store because its expected value (EV = $875,000) is greater than the expected value of the large store (EV = $250,000).

So the course of action will depend upon our forecast of the economy. If we think the economy will be strong, we will build the large store. If we think the economy will be weak, we will build the small store. We do not need to make a judgment at this point whether demand will be strong or weak, because that has been factored into the expected values.

We could move backward one more step and calculate an expected value for the overall decision, but in this case that would not be meaningful. We are really looking at two **mutually exclusive** projects at this point – either a large store will be built in the event of a strong economy or a small store will be built in the event of a weak economy. It would be meaningful to view the small store as giving us an option to expand to a large store in the event the economy is strong and the demand warrants it.

The above discussion makes it appear as though a decision tree can make the decision for us, which is an oversimplification. Only the decision-maker can make the final decision. In practice, a decision tree is merely a tool to aid discussion and support clear thinking about what's really going on. Furthermore, the probabilities associated with each of the outcomes in a decision tree are almost always **subjective**, meaning they are a best guess. Two different people might come up with two very different sets of probabilities, which could lead them to two very different conclusions.

The **benefits** of decision trees are:

1) They are helpful when there is a series of conditional choices,
2) They show the impact of time on decisions,
3) They can model uncertainty,
4) They produce quantitative results, and
5) They are flexible, examining the effects of predictors one at a time.

The **shortcomings** of decision trees are:

1) All decision factors must be expressed quantitatively. Qualitative factors are difficult, if not impossible, to express and utilize. For instance, how can you communicate customer goodwill or community image in terms of dollars?
2) Decision trees can be a challenge to develop in a group setting. Because of the frequently subjective nature of the probabilities associated with decision trees, developing and reaching agreement on event probabilities may be difficult.
3) There can be a great number of possible outcomes in the model, and the decision tree can become extremely large.
4) All data developed from decision tree analysis must be subjected to the good judgment of the decision-maker(s).

Sensitivity Analysis

Sensitivity analysis can be used to determine how cash flows can be expected to vary with changes in the underlying assumptions. Sensitivity analysis is a "what if" technique. Using expected cash flows, the NPV, IRR and PI of the project are determined. Then, the key assumptions that were used in making the original expected cash flow projections are identified. One assumption at a time is then changed, leaving the other assumptions unchanged; the NPV, IRR and PI are recalculated to determine what effect changing one assumption would have on those measures. This may show some area of risk that the company had not been aware of previously, and therefore indicate that the investment is riskier than originally thought.

Scenario Analysis

In **scenario analysis,** the NPV or IRR of a project is analyzed under a series of specific scenarios. These scenarios are based on macroeconomics, factors specific to the industry the firm operates in, and factors specific to the firm. Revenues, expenses and ratios under each of the scenarios are estimated, and the NPV and IRR of the project under each scenario are estimated. The decision to accept or reject the project is based on the NPVs and IRRs under all the scenarios, not just one.

Simulation Analysis

Simulation analysis allows for more than one uncertain element in the analysis. Therefore, simulation analysis is more comprehensive than **sensitivity analysis**. Simulations can be used to develop possible outcomes, using statistical methods and computing the NPV and IRR for each set of outcomes. Then, all of the results from all of the simulation runs are summarized into average, variance, coefficient of variation, etc., for all the statistics across all simulation runs. The final decision is based on the summary statistics. This is, however, an expensive method and will generally be used only with the larger projects.

Monte Carlo Simulation and "What-If" Risk Analysis

What-if analysis is a type of risk analysis that uses randomly generated values for probabilistic inputs. In risk analysis, we want to know both the probability of something happening such as a loss and, if it does occur, the potential magnitude of the loss. The analyst can estimate ranges for probabilistic inputs, such as labor costs or materials costs. Then, what-if analysis can be used to determine a **worst-case scenario** and a **best-case scenario**.

The base-case scenario is the most likely scenario, based on the analyst's estimates of the most likely probabilistic inputs. What-if analysis like this, which does not utilize simulation, can give us the various scenarios that may result, but it cannot tell us anything about the **probabilities** of the various scenarios.

Adding a **Monte Carlo simulation** to the model allows assessment of the probabilities of the various scenarios coming to pass, because we can generate random values for the probabilistic inputs based on the probability distribution for each probabilistic input. In addition to determining ranges for the probabilistic inputs, such as labor costs or materials costs, the analyst determines a probability distribution, the mean, and the standard deviation for each of these inputs. The computer simulation application generates the random values for the probabilistic input based on these inputs.

The values for the probabilistic input are then used to generate multiple possible scenarios. This is like performing statistical sampling experiments, except it is done on a computer and done over a much shorter time span than actual statistical sampling experiments. Enough trials are conducted (hundreds or thousands), using different values for the probabilistic inputs, to determine a probability distribution for the resulting scenario, which is the output. The repetition is an essential part of the simulation. If the simulation is run to evaluate the probability that a new product will be profitable, for instance, the output may include an average profit and the probability of a loss.

Benefits and Limitations of Simulation

Simulation is very flexible and can be used for a wide variety of problems. Simulation is easily understood, and thus management more readily accepts its results. Many simulation models can be implemented without special software packages, because most spreadsheet packages provide useable add-ins. For more complex problems, simulation applications are available.

However, simulation is not an optimization technique. It is a method that can predict how a system will operate when certain decisions are made for controllable inputs as well as when randomly generated values are used for the probabilistic inputs. Simulation can be effective for designing a system that will provide good performance; but there is no guarantee that it will be the best possible performance.

Furthermore, the results will be only as accurate as the model that is used. A poorly developed model or a model that does not reflect reality will provide poor results and may even be misleading. And there is no way to test whether the assumptions and relationships used are correct without the passage of time.

Other Techniques

Breakeven Analysis can be used to estimate the **revenue** that will be needed for a project to break even, in accounting terms. At the breakeven point, fixed costs + depreciation will be equal to revenue – variable costs.

Present Value Breakeven is the number of units a firm has to sell to arrive at an NPV of zero for the project.

The **informal method** takes the project that appears less risky. This may be used as the final decider in a case where two or more projects work out almost the same from a number standpoint. This is the gut-feeling method.

The **Capital Asset Pricing Model** assumes that all assets are held in portfolio rather than individually. In the portfolio the specific risk associated with each specific investment is eliminated because of the portfolio and therefore the only remaining risk is the market risk. As a result, the more sensitive an investment is to the market, the riskier that asset becomes.

Statistical Measurements of Cash Flow Variability

When forecasting cash flows for investment projects, we might make several sets of forecasts for each project to reflect the various alternative states of the economy that might ensue. If we are comparing two project proposals, we might make several forecasts for the cash flows expected in Year 1, as follows:

	Project A	Project B
Economy in a deep recession	$200,000	$100,000
Economy in a mild recession	250,000	200,000
Economy stable	300,000	300,000
Economy in a minor expansion	350,000	400,000
Economy in a major expansion	400,000	500,000

It is not hard to see that the expected cash flows for Project B under the possible states of the economy vary more than do the expected cash flows for Project A under the same states of the economy. This variability of cash flows in Project B makes it appear riskier.

However, this doesn't mean much to us yet. We need to quantify the analysis by determining the **probability** of each of the possible states of the economy, and then determine how much the cash flows are expected to vary in response to the different scenarios.

Suppose our economists tell us that the probability of a deep recession occurring next year is 5%; a mild recession, 10%; a stable economy, 50%; a minor expansion, 25%, and a major expansion, 10%. Now we can do something with this. First, we can calculate the **expected value** of the cash flows for Projects A and B:

	Probability	Project A		Project B	
	P	Cash Flow	CF × P	Cash Flow	CF × P
Economy in a deep recession	5%	$200,000	10,000	$100,000	5,000
Economy in a mild recession	10%	250,000	25,000	200,000	20,000
Economy stable	50%	300,000	150,000	300,000	150,000
Economy in a minor expansion	25%	350,000	87,500	400,000	100,000
Economy in a major expansion	10%	400,000	40,000	500,000	50,000
Expected Value			312,500		325,000

Now we have **expected** values for cash flows for Projects A and B next year. The **expected value** of the cash flows for each of the two projects is simply a **weighted average** of the possible cash flows, with the weights being the probabilities of each occurrence. This tells us that the expected value of the cash flow for the first year of Project B is higher than the expected value of the cash flow for the first year of Project A. However, it doesn't tell us which project is riskier.

The riskiness of each project can be inferred from the **dispersion**, or variability, of the distribution of the possible results. The usual method of expressing dispersion of results is the **standard deviation**. The narrower the distribution of the results, the lower the standard deviation will be – and the lower the risk will be. The wider the distribution of results, the greater the standard deviation will be and the greater the risk. So **standard deviation** is a measure of the **dispersion of a probability distribution** and thus a measure of the **riskiness of a project.**

Note:

- The **variability**, or **dispersion** of forecasted results is used to quantify risk.
- The **standard deviation** of data is used to describe the **dispersion** of data about its mean, or average/expected value. The narrower the distribution of the data, the smaller the standard deviation will be and the lower the risk. The wider the distribution of the data, the greater the standard deviation and the greater the risk.

A **normal distribution** has the shape of a bell. In a **normal distribution**, approximately 68% of the total area of the distribution will fall within **one standard deviation** of the **expected value**, on either side. So there is 32% chance (1 − .68) that an actual value will fall further than one standard deviation from the mean, or expected value. The probability that an actual value will fall within **two** standard deviations of the expected value of the distribution is approximately 95%; and the probability that a value will fall within **three** standard deviations is 99%. We can use **standard deviation to determine the likelihood that a particular event will occur.**

The **standard deviation**, or σ, of a probability distribution like those above can be calculated as follows:

- First, we take each possible cash flow in the series and subtract it from the **mean**, or **expected value** that we calculated above.
- We then square each difference and multiply the result by the probability of that cash flow's occurring.
- Next, we sum the results of those multiplications. This sum is the **variance**, which we will discuss later.
- Finally, we take the square root of the variance, which provides the standard deviation.

Example: The standard deviation of Project A 's Year 1 cash flow is:

Possible Cash Flows	Probability (P)	CF × P	(CF − Expected CF)² × P	
$200,000	.05	10,000	$(200,000 - 312,500)^2 \times .05 =$	632,812,500
250,000	.10	25,000	$(250,000 - 312,500)^2 \times .10 =$	390,625,000
300,000	.50	150,000	$(300,000 - 312,500)^2 \times .50 =$	78,125,000
350,000	.25	87,500	$(350,000 - 312,500)^2 \times .25 =$	351,562,500
400,000	.10	40,000	$(400,000 - 312,500)^2 \times .10 =$	765,625,000
Total		312,500		2,218,750,000

The Standard Deviation σ = the square root of 2,218,750,000, or 47,104

This means that the probability is 68% that the actual cash flow will be within ±$47,104 of $312,500, or between $265,396 and $359,604.

The Standard Deviation of Project B's Year 1 cash flow is:

Possible Cash Flows	Probability (P)	CF × P	(CF − Expected CF)² × P	
$100,000	.05	5,000	$(100,000 - 325,000)^2 \times .05 =$	2,531,250,000
200,000	.10	20,000	$(200,000 - 325,000)^2 \times .10 =$	1,562,500,000
300,000	.50	150,000	$(300,000 - 325,000)^2 \times .50 =$	312,500,000
400,000	.25	100,000	$(400,000 - 325,000)^2 \times .25 =$	1,406,250,000
500,000	.10	50,000	$(500,000 - 325,000)^2 \times .10 =$	3,062,500,000
Total		325,000		8,875,000,000

The Standard Deviation σ = the square root of 8,875,000,000, or 94,207

There is a much higher Standard Deviation for Project B. This means that the probability is 68% that the actual cash flow will be within ±$94,207 of $325,000, or between $230,793 and $419,207.

This means that Project B has a **greater dispersion of possible outcomes**. Therefore, we can say that **Project B has greater risk.**

The **square of the standard deviation**, σ^2, is called the **variance of the distribution**. In the above examples, Project A has a **variance** of $47,104^2$, or 2,218,786,816. Project B has a **variance** of $94,207^2$, or 8,874,958,849. The **variance of a population** is the average of the squares of the deviations of the measurements about their mean. When the concept of standard deviation is combined with probability as it is above, we are calculating a **weighted average** not only of the possible cash flows as we did first, but also of the squares of the variances. Thus, the **variance**, which in this case is a **weighted average**, is actually the weighted average of the squared deviations, which for Project A was 2,218,750,000, and for Project B was 8,875,000,000. (The small differences between the amounts above, calculated as the squares of the two standard deviations, and the amounts calculated in the example, are due to rounding.) The **variance of the distribution** is another indication of the riskiness of each project. The **variance** of Project B is greater than the **variance** of Project A.

The **coefficient of variation** is the ratio of the standard deviation of a distribution to the expected value of the distribution. The **coefficients of variation** for Project A and Project B are:

$$CV_A = \$47{,}104 \;/\; \$312{,}500 = .15$$

$$CV_B = \$94{,}207 \;/\; \$325{,}000 = .29$$

The coefficient of variation is greater for Project B than for Project A, which also means that Project B has a **greater degree of relative risk.**

So what do we do with this information now? We are considering two projects, Project A and Project B. Let's say for the sake of this discussion that they both entail the same initial investment and they are of the same length. Also, let's say they are mutually exclusive projects. If we choose A, we cannot choose B. If we choose B, we cannot choose A. The expected value of the cash flows from Project B for Year 1 – $325,000 – is greater than the expected value of the cash flows from Project A for Year 1 – $312,500. – Should we choose Project B because its expected cash flow is higher? Does the higher expected cash flow for Project B offset the increased risk we would be taking if we embark upon that project?

The answer to those questions is, of course, a judgment call that depends upon more information than we have here. But standard deviation as a measure of the variability of results can be used to quantify risk and thus assist in the decision-making process.

Adjustments to the Discount Rate for Risk or Inflation

Risk-Adjusted Discount Rate

Companies adjust for risk by using **risk-adjusted discount rates.** A company will **increase the discount rate** used in NPV calculations for more risky, or uncertain, investments. A higher discount rate will require higher expected future cash flows for the company to make the investment, thus making fewer investments acceptable. And it will lower the discount rate used for an investment that is judged less risky than the company's present portfolio of investments.

A company's Weighted Average Cost of Capital (WACC) – which is the rate-of-return required by investors in the company's securities – is the appropriate discount rate to use in capital budgeting decisions and NPV calculations **as long as the riskiness of the project is the same as the riskiness of the firm's existing business.** As we said earlier, for the risk premium to remain unchanged as a result of the capital expansion project, however, the following conditions must be met:

1) The new assets financed by the new capital must not change the firm's operating environment substantially. If any change is involved, risks as discussed above will enter into the equation.

2) The new capital must be raised in the same proportions as the existing capital, so that the firm's **financial risk** remains the same.

If either of the two assumptions does not hold true, the discount rate used to calculate NPV will need to be adjusted to reflect the changed risk profile of the firm as a result of the project under consideration.

If the new project involves going into a new market where there is stiff competition and the risk of failure is high, then the new project is going to change the firm's operating environment. The business risk of the firm will be increased. As a result, investors will require a higher rate of return to invest in either the debt or the equity of the firm, to compensate them for the increased risk they are assuming by making an investment in the company. The increase in the required rate of return is called the **risk premium**. When the risk premium increases, the company's overall cost of capital will increase.

Or, if the firm decides to use more debt to finance the new project than it has in its current capital structure, the amount of debt in its capital structure will increase. Up to a point, this is all right. However, if the proportion of debt in the company's capital structure becomes too high, this will make investors more nervous because of the greater financial risk of the company not being able to service its debt and going into default. Bond investors will require a higher rate of return, a risk premium again, to invest in the bond. The risk premium will increase, the rate of return the company will have to pay on its debt will increase, and that will cause the company's cost of capital overall to increase.

A higher discount rate is used in the capital budgeting analysis to reflect higher risk. When the discount rate is higher, **the expected cash flows from the investment will have to be higher in order to result in a positive NPV. If the expected cash flows from the investment are not** higher, increasing the discount rate could change a positive NPV to a negative NPV, which would cause the project, appropriately, to be rejected.

For example, if a project involves simply replacing an old machine with a new one, that will probably not change the firm's operating environment and therefore, it does not carry much, if any, risk. However, if, as we said above, the company is considering a new machine for the purpose of entering a new line of business, that project **will** involve uncertainty and therefore risk, because the outcome is unknown. We would probably use the Weighted Average Cost of Capital to discount the cash flows from the new machine that was simply replacing the old one. However, our required rate-of-return would be higher for the new machine if purchased for a new line of business, and we would therefore use a Risk-Adjusted Discount Rate.

Risk-Adjusted Discount Rate = Weighted Average Cost of Capital + Risk Premium

The amount of the risk premium is usually determined by management, based on the results of risk analyses such as those discussed above.

Note: Conversely, if the project is **safer** than the existing business of the firm, a discount rate that is **lower** than the present cost of capital should be used.

However, **discount rates should not be adjusted** for nonmarket risks that are unique or diversifiable, such as the possibility that a new drug may not be approved or that a drilling project will come up dry. Instead, the **cash flows should be adjusted** to reflect those risks. Project cash flows should give weight to all possible outcomes, favorable and unfavorable. Cash flows that do this will be correct **on average**. Sometimes the projections will be high and sometimes they will be low, but over several projects, they will average out. After adjusting cash flow forecasts for the **nonmarket** risks, **then** consider whether the chance of a bad outcome adds to the project's **market** risk.

The Capital Asset Pricing Model is frequently used to estimate the return on common stock that investors require. This is used to calculate the firm's Weighted Average Cost of Capital, which in turn is used to determine the risk-adjusted discount rate/required rate-of-return. There are various other models that can be used in addition to the Capital Asset Pricing Model, including discounted cash flow models with varying future growth rates and Arbitrage Pricing Theory. These were discussed in Section B in the topic of "Risk and Return." You may want to review them, as you may see them again in the context of capital budgeting.

Certainty Equivalent NPV

The Certainty Equivalent NPV can be **applied in conditions of uncertainty**, where a project's returns are risky. Rather than adjusting the discount rate, the certainty equivalent approach adjusts cash flows downward to levels that the company feels are certain. The NPV is then calculated using those cash flows and the risk free rate.

Certainty equivalent expected cash flow estimates are calculated by multiplying the expected cash flows for each period by a "certainty equivalent factor" for that period, ranging from 1.0 to 0. The certainty equivalent factors are assigned to each period's cash flows according to a subjective perception of the riskiness of the projected cash flows along with the risk aversion of the person assigning the certainty equivalent factors.

A certainty equivalent factor of 1.0 means the expected cash flow is certain. A certainty equivalent of 0 means the expected cash flow has the maximum amount of uncertainty, i.e., it is very uncertain. The certainty equivalent factors are set between 1.0 and 0, at a level such that the decision-maker is indifferent between receiving a higher risky return and a smaller certain return.

After each expected cash flow has been adjusted by its certainty equivalent factor, the project's NPV is calculated based on the adjusted cash flows, discounted at the **risk-free rate** rather than the company's required rate of return.

Inflation

As we have already discussed, in an **environment of inflation**, we need to make a couple of adjustments to this process.

1) First, the **discount rate to use should be increased**. This is because the market will require a higher rate-of-return to compensate for the increased risk of inflation. By **raising** the discount rate, we will **decrease** the present value of future cash flows and this will make the project less likely to have a positive NPV.

2) Secondly, we need to **increase the cash flow amounts in the future**. This is because **inflation will cause the dollar to be worth less** in the future, and the amounts of cash (both inflows and outflows) will therefore increase in the future.

   If inflation of 5% per year is expected, we would increase cash flows by 5% each year, **compounding the increase each year** to recognize the impact of inflation on the cash flows. These adjusted cash flows are called **nominal cash flows** in capital budgeting.

   The nominal cash flows projected in capital budgeting will be comparable to the actual cash flows ultimately recorded in the accounting system, since actual receipts and disbursements as recorded will reflect the effect of the inflation.

Real Options in Capital Budgeting

The idea of "real options" was developed in 1977 by MIT professor Stewart C. Myers. Myers took the concept of financial options – American call options in particular – and applied it to capital budgeting under conditions of uncertainty. The **real options approach** is viewed as a problem of optimization of a real asset (a piece of equipment, a building, land, a project, etc.) under uncertainty, given the available options.

To analyze a proposed capital budgeting project using NPV, it is necessary to make estimates of expected future cash flows and an appropriate discount rate. But with NPV, we can utilize only information that is known at the time these estimates are made, and the choice is all or nothing. The NPV approach ignores the fact that most investments can be delayed, and that the option to delay can have value. Real options, on the other hand, are not about simply calculating a single NPV before the project begins, then making a decision to accept the project and sitting back passively until the project's term runs out. Instead, real options provide a framework for strategic decision-making as the project goes along. The choice is an initial choice, followed by more choices as more information becomes available.

In a FASB Special Report, "Business and Financial Reporting, Challenges from the New Economy," by Wayne S. Upton, Jr., Upton writes:

> "A *real option* is easier to describe than to define. A financial option is a contract that grants to the holder the right but not the obligation to buy or sell an asset at a fixed price within a fixed period (or on a fixed date).[f] The word *option* in this context is consistent with its ordinary definition as "the power, right or liberty of choosing."[g]

Thus, a real option is the right, but not the obligation, to acquire the gross present value of future expected cash flows by making an investment on or before the date the opportunity expires.

Upton continues:

> "Real option approaches attempt to extend the intellectual rigor of option-pricing models to valuation of nonfinancial assets and liabilities. Instead of viewing an asset or project as a single set of expected cash flows, the asset is viewed as a series of compound options that, if exercised, generate another option and a cash flow.[h]
>
> Proponents argue that the application of option pricing to nonfinancial assets overcomes the shortfalls of traditional present value analysis, especially the subjectivity in developing risk-adjusted discount rates. They contend that a focus on the value of flexibility provides a better measure of projects in process that would otherwise appear uneconomical."[i]

Users of the real options approach to dealing with risk in capital budgeting view a capital budgeting investment opportunity as similar to an American call option, with the exercise price being the investment amount and the underlying being the project. Investing itself may create new options, such as the option to abandon a project or the option to expand it.

Real options analysis can enable a company to make an initial investment, and then proceed – or not – as the project develops. Thus, it is possible that based on real options analysis, a company might undertake a project with a negative NPV because it could offer expansion opportunities or because the project could be abandoned if things were to go bad. In the second scenario, the company could possibly restart the project later if the chances of success appeared to be greater at that time. These options can be valued; and it is possible that a negative NPV project would be undertaken because of the value of its options.

[f] Wayne S. Upton, Jr., Business and Financial Reporting, Challenges from the New Economy (Norwalk, Connecticut: Financial Accounting Standards Board, 2001), 92.

[g] Ibid, citing Webster's New World Dictionary, 3d. college ed. (New York: Macmillan General Reference, 1994), 951.

[h] Upton, Business and Financial Reporting, Challenges from the New Economy, 92.

[i] Ibid, 92 - 93.

Here are a few common real options:

- **The option to make follow-on investments if the immediate investment project succeeds.** For example, suppose a company is evaluating an investment in a new $100 million plant to manufacture a newly developed product, but the project would require very large sales to result in a positive NPV. A real option could be to build a smaller plant instead for only $10 million, and wait and see if the new product turned out to be a blockbuster. If so, then the $100 million plant could be built. In this case, the cost of the option is $10 million. The company is acquiring a real option to expand while obtaining strategic "first-mover" (first into a market) advantage.

- **The option to abandon a project.** If actual cash flows turn out to be much less than forecasted, it is good to have the option of bailing out and recovering the investment in the project by selling it. If the abandonment value of the assets is greater than the present value of the future cash flows from continuing the project, the project can, and very possibly should, be abandoned. Thus, the option to abandon a project is comparable to a put option on a financial asset.

  Furthermore, a project may be abandoned only temporarily if, for instance, actual cash flow is below forecasted cash flow because of temporary market conditions. When market conditions improve and prices rebound, the project can be revived.

  When the ability to abandon exists, the value of a project may be greater than if the abandonment option were not available. Furthermore, the recognition of this option can have a significant effect on project selection in a situation of capital rationing.

- **The option to wait and learn more before investing.** A real options approach can be taken to find the optimal timing of an investment. Traditional capital budgeting assumes either immediate acceptance or immediate rejection of a project. However, those are not the only choices. If a project's forecasted cash inflows are large, the company will probably want to invest without delay in order to capture those cash flows. But if the forecasted cash flows are small, managers may be more inclined to wait to invest, even if the NPV of the project is positive. A choice could be the option to wait another year to learn more about the market for the proposed project.

  Sometimes, particularly with real estate developments, the wait is a long one. This is because once the land is developed, its use will be limited to the purpose for which it was developed due to the expense involved in changing its use. By waiting, the owner of the land can observe changes in values of developed properties in the same neighborhood and make better estimates of expected cash flows from alternative investments. As time passes, expected cash flows from one of the investment alternatives may emerge as being significantly higher than the others.

  The greater the variability in possible outcomes, the greater the value of the option to wait and learn.

- **The option to vary the inputs to the production process, the production methods, or the firm's output or product mix.** Equipment can be designed to operate in different ways or with different raw materials, depending upon the specific conditions. Production can be shifted from one product to another to adapt to changing market demands. Even if this shift in production increases production cost, it can result in increased cash flow if the alternative would be production of a product that is not marketable because the demand for it is not sufficient.

Upton references other options as well:

- Growth options, as in the decision to invest in entry into a new market, and
- Flexibility options, as in the choice between building a single, centrally located facility or building two facilities in different locations.[J]

[J] Upton, Business and Financial Reporting, Challenges from the New Economy, 92, citing Martha Amram and Nalin Kulitilaka, Real Options: Managing Strategic Investment in an Uncertain World (Boston: Harvard Business School Press, 1999), 10.

Decision Trees and Valuing Real Options

> Note: The calculations in the examples that follow are presented for your understanding only. Calculations like these **will not** be required on the exam. However, you may need to know that the value of a real option is the difference between the Net Present Value of the project **with** the real option and the Net Present Value of the project **without** the real option.

Decision Trees are frequently used to quantify the real options inherent in a project. Companies use Decision Trees to identify future choices available, understand the risk involved and assign probabilities, and clarify how future decisions will affect project cash flows.

Suppose a company is considering investing in a new product. The management believes there is a 50% chance the demand for the product will be strong, and a 50% chance the demand will not be strong. The company can manufacture the product on a small scale for an initial investment of $150,000 using its current facilities, and introduce the product in a few test markets to see if demand is strong. If the demand is strong, management expects cash flow in the first year of $250,000. If this occurs, they will plan to build a new factory at a cost of $10,000,000 and roll out the product nationwide. If the demand is not strong, cash flow in the coming year will be only $100,000. If this occurs, they will not build the new factory and will discontinue the product, shifting the existing facilities to manufacture of other product lines.

If the company evaluates this plan for the first year using traditional discounted cash flow analysis and a required rate of return of 10%, the expected value of the projected cash inflow in Year 1 is (.50 × $250,000) + (.50 × $100,000), or $175,000. Discounted at 10%, the present value of $175,000 is $175,000 × .90909, or $159,091. With an initial investment of $150,000, this project has an NPV of $9,091.

This company has, in effect, acquired an option to expand. In one year, management will evaluate the market response to the product and make their decision whether to expand or not.

In one year, if the first year's cash inflow from the project is $250,000, then the company will project that demand for the new product is strong and it should begin building the new factory for $10,000,000. If the company does build the new factory at the end of Year 1, cash inflows will begin at the beginning of Year 3 (construction will take a year). Here are management's projections for expected cash inflows from the new factory's output, beginning with Year 3 and continuing to Year 7:

Year 3	**2,000,000**
Year 4	**4,000,000**
Year 5	**6,000,000**
Year 6	**9,000,000**
Year 7	**10,000,000**

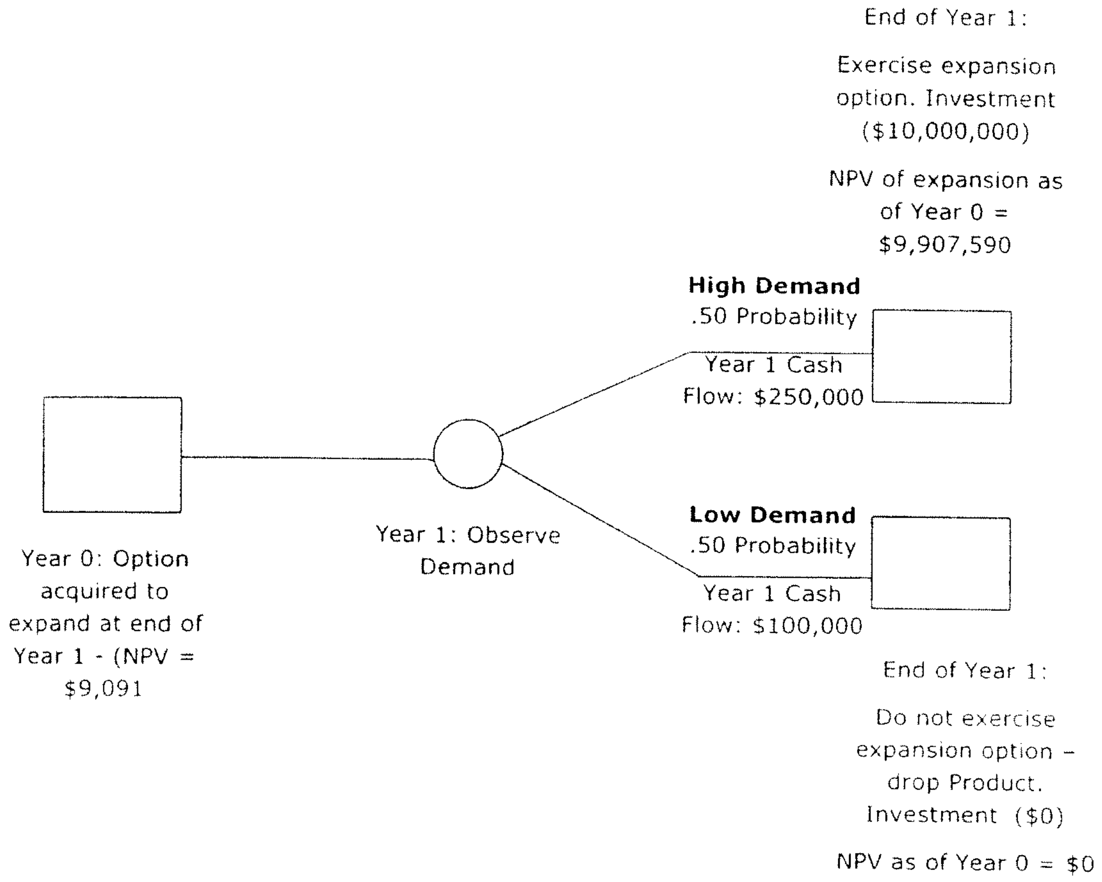
Remember that Year 3 for this analysis of the expansion is like Year 1 of a traditional capital budgeting analysis, and so Year 2 is like Year 0. Thus, the Present Value of the cash inflows **as of the end of Year 2 and beginning of Year 3** will be the following:

(2,000,000 × .90909) + (4,000,000 × .82645) + (6,000,000 × .75131) + (9,000,000 × .68301) + (10,000,000 × .62092) = 21,988,130. Subtracting the investment made during Year 2 (assumed to be made at the end of Year 2) for the construction of the new plant, we get an NPV **for the expansion only** of (21,988,130 – 10,000,000), which equals $11,988,130.

Since this NPV for the expansion is as of the end of Year 2, we need to discount it again for two years to Year 0. The Present Value of $1 factor for 2 years at 10% is .82645, so the Present Value of this Net Present Value of $11,988,130 is $9,907,590.

Following is their Decision Tree. Each box represents a management decision. The circle represents an outcome that is not yet known.

An Expansion/Contraction Option Expressed as a Decision Tree



Valuing the Real Option

Because options limit the downside potential of a project, the value of a real option is **increased** when the uncertainty relating to its underlying asset is greater. Thus, the greater the number of options and the greater the uncertainty surrounding their use, **the greater will be the worth of the project.**

To value a real option, **the first step is to value the project as if it had no options attached.** We did this in the example above, when we arrived at an NPV of $9,091. This is the way the Net Present Value of the project is determined, with no provision for real options.

Then, the various options and possible results are set up on a Decision Tree, as we have done above, using the various possible outcomes, and there could be more than 2 possible outcomes. **The expected value of the option(s) is determined by using this Decision Tree and probabilities of each event occurring to determine the payoffs under each possible combination of events.** The possible events may be permanent abandonment, temporary abandonment, varying inputs or outputs, varying the production mix, or any of a number of other possibilities.

In our example above, we have a probability of 50% of having an NPV of $9,907,590 for the expansion, and a probability of 50% of having an NPV of zero. The expected value of this expansion/no expansion option is thus (.50 × $9,907,590) + (.50 * 0) = **$4,953,795, and this is the value of the real option.**

The net present value of the project **with** the real option, then, is the value of the project without the real option to expand, which is $9,091 **plus** the value of the real option, which is $4,953,795, for a total project worth of $4,962,886.

By starting out on a small scale, the company will have an opportunity to test the market, iron out possible design flaws and make a decision whether to expand to full-scale production.

The value of a real option is the difference between the Net Present Value of the project with no real option, and the Net Present Value of the project with the real option.

The worth of a project is the Net Present Value of the project with no real options, plus the value of the real options.

Project Worth = NPV + Real Options Value

The value of a real option can also be determined by calculating the net present value of the project **without** the real option, then calculating the net present value of the project **with** the real option, and then finding the difference. This will be illustrated in our next example.

The Value of an Abandonment Option

Abandoning a project may include selling the project's assets or employing them in another area. Whether the assets would be sold or whether they would be re-employed, an abandonment value can be estimated and included in the value of the project as a real option. The ability to abandon an investment project can enhance the worth of the project. Some assets, of course, have no abandonment value because they have no market value or alternative use, and for those assets, the abandonment value is zero.

However, funding should be withdrawn from a project and the project abandoned whenever the project can no longer justify the continued use of the funds. An investment project should be abandoned when

1) its value as an abandoned project is greater than the present value of the project's future cash flows, and
2) it is better to abandon the project now than at some later date.

Probability Concepts Review

Before we continue, we should review probability concepts, because this next example will require the use of probability. When we are estimating future cash flows, we develop several possible cash flows and we also develop the probability of each possible cash flow's occurring. When we value real options, we need to use these probability concepts, because real options involve unknowns which we quantify with probabilities.

Note: In the previous example, we did not provide probabilities. However, probabilities would have been used by management in developing the expected cash flows which we referenced, because **expected cash flow** is always a weighted average of the possible cash flows, weighted according to each possible cash flow's probability of occurring.

Suppose we were to say, "The probability of cash flow being $1,000,000 in Year 1 is .30. The probability of cash flow being $1,500,000 in Year 2, **if** cash flow is $1,000,000 in Year 1 is .40. Therefore, the probability of cash flow being $1,000,000 in Year 1 **and** $1,500,000 in Year 2 is .12."

We have three types of probability in these statements:

1) The probability of cash flow being $1,000,000 in Year 1 (which is .30) is called the **initial probability.**
2) The probability of cash flow being $1,500,000 in Year 2 **if** cash flow is $1,000,000 in Year 1 (which is .40) is called the **conditional probability** because it is conditioned upon cash flow first being $1,000,000 in Year 1.
3) The probability of cash flow being $1,000,000 in Year 1 **and** $1,500,000 in Year 2 (which is .12) is called the **joint probability** because it is the probability that both events will occur.

The **initial probability** multiplied by the **conditional probability** equals the **joint probability**.

Initial Probability × Conditional Probability = Joint Probability

The Abandonment Option Example

We are considering building a new plant to manufacture a new model of automobile, the Sipper, which is a hybrid automobile (it runs on both gasoline and electricity). We know that the Sipper will be manufactured for no more than two years as it is currently designed, because a new model that runs on hydrogen only will be introduced as the Hydro-Sipper in two years. The Hydro-Sipper will require completely different manufacturing facilities.

Furthermore, it is possible that the Sipper will be produced for only **one** year. If, after one year, the abandonment value of the plant is greater than the present value of the expected future cash flows for the second year, then the plant will discontinue manufacturing Sippers and the facilities will be abandoned.

For the sake of simplicity, we will assume that, at the end of **two** years, there will be no further cash flow from the Sipper under any circumstances; and furthermore, the plant will have no residual value at that point in time. However, if we decide to abandon the project after only **one** year, the plant will have an expected abandonment value of $2.5 million.

The plant will cost $5 million. The company's required rate of return is 10%. The probabilities of various possible cash flows for the first year that management has established are as follows:

Probability	Cash Flow
.25	$2,000,000
.50	$4,000,000
.25	$6,000,000

The probabilities above for the first year are the **initial** probabilities. Management has determined that if cash flow is $2,000,000 the first year, another set of three probabilities will apply for the second year; if cash flow for the first year is $4,000,000, a different set of three probabilities will apply for the second year; and if cash flow for the first year is $6,000,000, still another set of three probabilities will apply for the second year. These three sets of three probabilities for the second year are the **conditional** probabilities. We will set these up in a table and show the **joint probabilities,** as follows. You should recalculate the joint probabilities to make sure you understand where they come from. Note also that the joint probabilities total to 1.0000, or 100%.

YEAR 1		YEAR 2		
Initial Probability	Cash Flow (in 000's)	Conditional Probability	Cash Flow (in 000's)	Joint Probability
		.25	$1,000	.0625
.25	$2,000 →	.50	$2,000	.1250
		.25	$3,000	.0625
		.25	$3,000	.1250
.50	$4,000 →	.50	$4,000	.2500
		.25	$5,000	.1250
		.25	$4,000	.0625
.25	$6,000 →	.50	$6,000	.1250
		.25	$7,000	.0625

				1.0000

In developing this capital budgeting analysis (without the real option), our first year **expected cash flow** will be:

(.25 × $2,000) + (.50 × $4,000) + (.25 × $6,000) = $4,000.00

Our second year **expected cash flow** will be:

(.0625 × $1,000) + (.125 × $2,000) + (.0625 × $3,000) + (.125 × $3,000) + (.25 × $4,000) + (.125 × $5,000) + (.0625 × $4,000) + (.125 × $6,000) + (.0625 × $7,000) = $3,937.50

We now have all the information we need to develop the NPV of this project **without the real option**. We will discount the first and second years' expected cash flows to their present values using the company's required rate of return of 10%, and then calculate the net present value, as follows:

Year 1 Expected Cash Flow:	$4,000.99	×	.90909	=	$ 3,636
Year 2 Expected Cash Flow:	$3,937.50	×	.82645	=	$ 3,254
Less: Initial Investment					(5,000)
NPV					$ 1,890

However, this NPV will be quite different when we incorporate into the analysis the effect of potentially abandoning the project after the first year.

Remember we said that the abandonment value at the end of Year 1 would be $2.5 million. If Year 1's cash flow is only $2 million — and there is a 25% chance it will be — then the expected cash flow for Year 2 will also be $2 million. This is calculated using the three conditional probabilities and cash flows that follow the Year 1 cash flow of $2 million: (.25 × $1,000) + (.50 × $2,000) + (.25 × $3,000). This Year 2 cash flow of $2 million is less than the plant's abandonment value of $2.5 million. Therefore, if the Year 1 cash flow is only $2 million, this project will be abandoned at the end of Year 1.

If the project is abandoned after Year 1 — and there is a 25% probability that it will be — the Year 1 expected cash flow associated with that initial probability will be increased by the expected abandonment value of $2.5

million, from $2 million to $4.5 million. The conditional probability following that initial probability will be 1.0 or 100%, and the expected cash flow for Year 2 associated with that conditional probability will be zero.

So here is the revised schedule of possible cash flows and their probabilities, when we incorporate the potential abandonment of the plant into the table of probabilities:

YEAR 1		YEAR 2		
Initial Probability	Cash Flow (in thousands)	Conditional Probability	Cash Flow (in thousands)	Joint Probability
.25	$4,500	1.00	$0	.2500
.50	$4,000	.25	$3,000	.1250
		.50	$4,000	.2500
		.25	$5,000	.1250
.25	$6,000	.25	$4,000	.0625
		.50	$6,000	.1250
		.25	$7,000	.0625
				1.0000

Now, our expected cash flows for our capital budgeting analysis, **with the real option** of abandonment, are as follows:

Year 1: (.25 × $4,500) + (.50 × $4,000) + (.25 × $6,000) = $4,625.

Year 2: (.25 × $0) + (.1250 × $3,000) + (.25 × $4,000) + (.125 × $5,000) + (.0625 × $4,000) + (.125 × $6,000) + (.0625 × $7,000) = $3,437.50.

And here is our capital budgeting analysis, **with the real option**, using these revised expected cash flows:

Year 1 Expected Cash Flow:	$4,625.0	×	.90909	=	$4,205
Year 2 Expected Cash Flow:	$3,437.5	×	.82645	=	2,841
Less: Initial Investment					(5,000)
NPV					$2,046

The NPV **with** the real option is $2,046,000. The NPV **without** the real option is $1,890,000. The **difference of $156,000 is the value of the real option.**

Decision Trees and tables such as the ones above can become quite complex. For example, a Decision Tree could be used to illustrate the results of two different decisions under varying levels of demand, the probabilities of each of those levels of demand, the courses of action that might be taken under each level of demand, probabilities of success for each of those courses of action and the payoff under each scenario. For a project containing several real options, the expected cash flow is a combination of all the probabilities and all the possible cash flows, starting at the right side of the tree and working backward. The Project Worth, or the Net Present Value of the investment incorporating all the real options and possible future cash flows, can be calculated in this way.

Thus, Decision Trees can be used to understand and quantify the links between today's decisions and tomorrow's decisions. However, because they can be so complex, they should not necessarily be comprehensive but rather limited only to the most important links.

Monte Carlo Simulation

As an alternative to the decision tree, **Monte Carlo** analysis may be used to determine the Net Present Value of the project with the real options by building all of the possible payoffs under the real options into the Monte Carlo analysis model. The result is an averaged approximate NPV with the real options.

A Monte Carlo simulation allows the decision-maker to consider **all possible** combinations of project outcomes by using a computer to simulate the possible outcomes. In capital budgeting, this simulation utilizes a model where all the variables are defined: market size, product price, market share, unit variable cost and fixed cost. Then the probabilities of each possible outcome for each variable are specified. The effect of all the possible events on subsequent years' results is determined. All this is built into the model. Then the computer creates random scenarios and calculates the resultant cash flows for each period. After multiple iterations, an estimate of the probability distributions of the project's cash flows emerges. The accuracy of the estimate will depend upon the accuracy of the model and the interrelationships between the variables.

The probability distributions of the cash flows enable calculation of expected cash flows. Then, those expected cash flows can be discounted to find their present value. Several NPVs are calculated based on the random choices of variables. The NPVs are averaged to get an approximate NPV for the project.

However, because a Monte Carlo simulation tends to emphasize expected value, it can be less than realistic if market growth, market share and costs, etc. diverge from expected levels.

Other Methods of Valuing Real Options

Real options can also be valued similarly to the way that an American call option on a stock is valued. An American call option on a stock gives the holder the right but not the obligation to purchase the stock at a set price, which can be exercised at **any time before its maturity date.** (In contrast, a European option can be exercised only **on its maturity date.**) When an investor buys a call option, he or she is taking a position in the stock but is putting up less money than would be necessary to purchase the stock outright. An option has a higher beta and a higher standard deviation of return than a stock; and thus, an option is riskier than the underlying stock. A call option's risk is related to the price of the stock relative to the exercise price of the option, and the option's risk changes each time the stock price changes.

If the price of the underlying stock rises above the exercise price of the option, the option is worth the price of the stock minus the exercise price of the option. If the price of the stock falls below the exercise price of the option, the call option is worthless, and the investor's loss is the amount paid for the option, rather than the amount the investor would have paid to purchase the underlying stock.

The Black-Scholes Option Pricing Model

One means of valuing a call option is the Black-Scholes Option Pricing Model, which combines a common stock investment with a loan to create an **option equivalent**. The value of the option then equals the net cost of buying the option equivalent.

Fisher Black and Myron Scholes noticed that the values of an option at different price levels of the stock could be approximated by borrowing money to purchase the actual stock rather than purchasing the option. They determined that the option had the same value as a stock purchase net of the present value of the loan. Since the stock and the loan can be valued, the option can also be valued.

The Binomial Method for Valuing Options

The Binomial Method for valuing options incorporates all of the factors that impact option values by assuming that one of only two outcomes will occur in each period: an upside or a downside. (It is called the "Binomial Method" because of the limit on outcomes to only two possibilities.) It values options by forming a risk-free "twin" portfolio from which the outcomes can be discounted using the risk-free rate. To create a more realistic distribution of possible stock prices, the period can be divided into shorter and shorter intervals, with two possible prices for the stock in each interval, resulting in a continuum of possible future stock prices. The Binomial Method also involves a stock purchase and a loan for the purchase, just as the Black-Scholes Method does; and the value of the option is equal to the value of the stock purchase and net of the present value of the borrowing.

Comparison of the Black-Scholes Method and the Binomial Method

The Black-Scholes model of valuing options recognizes a continuum of possible outcomes per period, whereas the Binomial model recognizes only two possible outcomes in each period. The Binomial Method also adds an assumption that investors are risk-neutral.

As the number of intervals analyzed increases (i.e., each interval becomes shorter), the option value obtained from the Binomial Method and that obtained from the Black-Scholes method approach one another.

Real options involve complex issues and trade-offs. In valuing real options, each real investment opportunity can be thought of as having a "double," a stock or a whole portfolio of stocks that have the same risk as the investment project. A key assumption is that an investor would be willing to pay the same amount for a **real option** based on the project as he/she would for a traded option on the "double" investment.

The value of real options does not show up anywhere on the balance sheet of a company. However, investors know they are there and include them in their valuation of the company's stock. If a company has valuable real options, the market value of its stock will be higher than the market value of the physical assets on its balance sheet.

The Qualitative Factor in Capital Budgeting Decisions

At the end of the day, what influences the final decision in a capital budgeting analysis may not be any of the quantitative methods discussed above. There may be qualitative factors that override any quantitative "go" or "no go" decision that quantitative methods can produce. Some of these qualitative factors are:

- The investment might improve the quality of products and services offered.
- The investment might shorten the time in which products and services can be produced and/or delivered to customers.
- The investment might address consumer safety concerns.
- The investment might be required because of government regulations or environmental protection concerns.
- Worker safety might be improved by the investment.
- The company's public relations – its image and prestige – might be impacted positively by the project.
- The community where the firm operates could be served by the investment.
- The owners and/or the management might simply want to make the investment.

Valuation

Valuation of Stock

We discussed the price of a share of stock earlier in the context of a company's cost of capital. The same techniques can be used by an investor to value an investment in order to make an investment decision.

Value of Preferred Stock

Recall that the value of a share of preferred stock is

$$P_0 = \frac{\textbf{Annual Dividend}}{\textbf{Investors' Required Rate of Return}}$$

This is also the value of a share of common stock if the dividend is not expected to grow.

The Importance of Growth in Valuation

Growth, by itself, does not create value and increased wealth for stockholders. Growth, **combined with investment in projects that return profits in excess of the capital invested**, creates value.

Valuing Common Stock Using the Dividend Growth Model or Constant Growth Model

In the section on the cost of capital, we said that the cost of retained earnings was calculated using the following formula:

$$C_{re} = \frac{d_1}{P_0} + g$$

Where: C_{re} = Cost of retained earnings, expressed as a percentage, which is the investors' required rate of return

d_1 = The **next** dividend that is to be paid (last year's dividend multiplied by 1 + the annual expected % growth in dividends)

P_0 = Common stock price today

g = The annual expected % growth in dividends

This formula goes by many names. It can be called the Dividend Growth Model, the Dividend Discount Model, the Gordon Growth Model or the Constant Growth Dividend Model.

The formula can be restated to solve for P_0 instead of for the investors' required rate of return and used to calculate the value of a share of common stock when the dividend is expected to grow at a stable rate.

The key variables needed in order to use the Dividend Growth Model to calculate the value of a share of stock are

- **d_1**, the ***next*** annual dividend to be paid;
- **r**, the investors' required rate of return; and
- **g**, the expected growth rate of the dividend.

The restated Dividend Growth Model formula is:

$$P_0 = \frac{d_1}{r - g}$$

Where:

- P_0 = the fair value today of a share of stock;
- d_1 = the ***next*** annual dividend to be paid;
- r = the investors' required rate of return; and
- g = the expected growth rate of the dividend.

> Remember that the Dividend Growth Model assumes that the dividends on a share of common stock will grow at a constant rate. If the dividend is **not** expected to grow, we can still use the formula above, but we must use **zero** for the "g" in the formula. Then we simply have a model that is similar to the valuation model used for preferred stock: Next Annual Dividend / Investors' Required Rate of Return.

Use of the Capital Asset Pricing Model With the Dividend Growth Model in Valuing Stock

One of the necessary variables for the Dividend Growth Model is the value of r, the investors' required rate of return. Remember that the Capital Asset Pricing Model is used to calculate the investors' required rate of return.

Therefore, if the investors' required rate of return is not known, the CAPM formula can be used to determine the investors' required rate of return, and then that required rate of return can be used in the Dividend Growth Model to determine the market value, or the price, of a share of common stock.

The key variables for the CAPM are:

- r_f, the risk-free rate;
- r_m, the market rate of return; and
- β, the stock's beta coefficient;

The CAPM formula is:

$$r = r_F + \beta(r_M - r_F)$$

The result, **r,** is the investors' required rate of return.

The theory behind the CAPM is that **investors will price investments so that the expected return on a security or a portfolio will be equal to the risk-free rate plus a risk premium** proportional to the risk, or beta, for that investment.

($r_M - r_F$) is the **market risk premium.** It measures the additional return (above the risk-free rate) that investors demand in order to move investments into the stock market in general (not to buy a given stock specifically). The stock market is generally riskier than the bond market.

The **beta coefficient (β)** represents the correlation between the expected returns of **a given stock** vs. the expected return of the average stock in the market as represented by some index of market activity such as the S&P 500.

$\beta(r_M - r_F)$, or the beta coefficient for a particular security multiplied by the market risk premium, is **the risk premium for that particular security**. It is the risk premium that investors require to purchase **that stock**.

If we have enough information to use the CAPM, we can use it to calculate the investors' required rate of return, then take the required rate of return and plug it into the Dividend Growth Model (restated to solve for P_0) to calculate the value of a share of stock.

Example: ABC Corporation's historical beta is .8. The risk-free rate of return is 3%. The market return is 5%. ABC Corporation paid a dividend last year of $1.00 per share. Dividends are expected to grow at a rate of 2% per year. What is the value of a share of ABC Corp. stock?

First, we must compute the investors' required rate of return using the CAPM:

$$r = r_F + \beta(r_M - r_F)$$
$$r = .03 + .8(.05 - .03)$$
$$r = .046 \text{ or } 4.6\%$$

Now, we can calculate the value of a share of stock using the Dividend Growth Model using the investors' required rate of return that we just calculated as r in the CAPM:

Remember that the dividend we need to use is the **next** dividend, not the past year's dividend. Last year's dividend was $1.00 per share, and dividends are expected to grow at a rate of 2% per year. Therefore, the **next** dividend amount to use in this equation is the dividend expected to be paid this year, which is $1.00 × 1.02, or $1.02.

$$P_0 = d_1 / (r - g)$$
$$P_0 = 1.02 / (.046 - .02)$$
$$P_0 = \underline{\underline{\$39.23}}$$

Note: The meaning of "next dividend" causes some problems. The next dividend is the annual dividend expected by the end of the coming year. That does not mean fiscal year or calendar year. It means the total annual dividend expected during the 12 month period **beginning today**.

- If **last year's** dividend is given, it should be increased by the annual growth rate by multiplying it by 1 + the growth rate to calculate the **next dividend.**
- If **this year's** anticipated dividend is given, it should be considered to be the **next dividend** and used as given without increasing it.

The Two-Stage Dividend Discount Model

The model above assumes that a constant growth rate in dividends will continue indefinitely. This may be true for some mature companies, but it is frequently not an appropriate assumption. Sometimes a company is going through a "growth spurt" where it is growing rapidly. That rapid growth is expected to last for a few years and then slow down to a more normal growth rate. In this situation, the constant growth dividend model must be adjusted.

We adjust the model by dividing the projected dividend cash flow stream into two parts: (1) the initial fast growth period, and (2) the next period, when normal and sustainable but lower growth is expected.

It is easier to demonstrate how this is done than to explain it, and it is easier to understand an example than it is to understand an explanation. So here is an example.

Example: Fastgrowth Industries is growing, and its dividend is growing, at an annual rate of 20%. This growth is expected to continue for three years into the future, after which the growth is expected to slow down to a more sustainable growth rate of 7%. Investors' required rate of return is 15%. The next annual dividend is expected to be $1.00.

Step 1: We will first calculate the present values of the dividends to be received during the first three years and sum the results. Each year's dividend increases over the previous year's dividend by 20%.

End of Year	Dividend	PV Factor @ 15%	Present Value of Dividend	
1	$1.00	.86957	$ .870	
2	1.20	.75614	.907	
3	1.44	.65752	.947	
Total PV of future dividends - Years 1 through 3:			**$2.724**	**or $2.72**

Step 2: We next project the dividend for Year 4 by multiplying the Year 3 dividend by (1 + the growth rate for Year 4), which is 1.07. Remember that growth is expected to slow down to 7% in Year 4. The Year 4 dividend is therefore projected to be $1.44 × 1.07, or $1.54.

We now pretend that Year 4 is Year 1 and so the end of Year 3 is Year 0. We use the Constant Growth Model to calculate what the value of the stock will be at the end of Year 3, assuming a required rate of return of 15% and an annual growth in dividends of 7% going forward from the end of Year 3, beginning with Year 4:

$$P_3 = d_4 / (r - g)$$

$$P_3 = \$1.54 / (.15 - .07)$$

$$P_3 = \$19.25$$

However, this present value of $19.25 occurs at the end of Year 3, not at Year 0. Therefore, it needs to be discounted back 3 years to Year 0. We will discount it back as if it is a single sum that will be received in 3 years. The present value factor for 3 years at 15% is .65752, so the present value of $19.25 three years from now is $19.25 × .65752, or **$12.66.** This is the present value as of Year 0 of the dividends to be received beginning at the end of Year 4 and continuing indefinitely.

Step 3: The final step is to add together the present value of the future dividends for Years 1 through 3 (**$2.72**) and the present value of the dividends to be received from Year 4 to infinity (**$12.66**) to calculate the value today, at Year 0, for a share of this stock:

$2.72 + $12.66 = $15.38

$15.38 is an appropriate market price for this stock, given the projected dividends and the 15% required rate of return by investors in the stock.

Other Valuation Methods – The P/E Model

A simple method of valuing a common stock is by calculating its P/E, or Price/Earnings, ratio. The P/E ratio is the price per share of common stock divided by the earnings per share. The P/E ratio is an indication of how much investors are willing to pay for a stock, for each dollar of the company's earnings.

A P/E ratio that is higher than the average P/E ratio of all stocks is usually an indication that investors expect the stock's earnings to increase. It may also indicate that investors perceive low risk in the stock. The P/E ratio can be used to value a business that is not publicly owned, when there is no market price for the stock. P/E ratios of public companies in the same industry are reviewed to determine an appropriate P/E ratio for companies in that industry. Then, that P/E ratio may be adjusted for the amount of risk the non-public company is perceived to have, compared with the public companies. The P/E ratio is adjusted downward for a higher risk company and upward for a lower risk company. Finally, the earnings per share of the non-public company is multiplied by the adjusted P/E ratio to calculate an appropriate stock price for the non-public company.

Example: Privately-held NMP Corporation has earnings per share of $3.00. After reviewing the P/E ratios of public companies in the same industry as NMP, we have determined that they sell for between 10 and 15 times earnings. Thus, an appropriate stock price for NMP would be between ($3 × 10) and ($3 × 15), or between $30 and $45 per share. Next, we evaluate the amount of business risk NMP has, compared with the business risk for other companies in the same industry. We determine it is on the high end of the risk profile. Therefore, the price for the stock should be in the low end of the range, to compensate for the increased risk. We therefore calculate that an appropriate price for the NMP Corporation stock would be $32 per share.

This valuation technique is similar to what is done when a business is considering purchasing a company that is not public, as we will see in the following topic.

Valuing Businesses, Business Segments, and Business Combinations

There are two commonly used methods of valuing a business:

1) The **market multiple** approach, sometimes called the comparative sales approach; and
2) The **discounted cash flow** approach, also called the income approach.

The Market Multiple Approach to Valuing a Business

The market multiple approach to valuing a business for investment purposes uses the market value of businesses that are similar to the business being valued. The values of the other businesses may be determined either by the trading price of publicly-owned companies or by the purchase price in a sale of a similar business that has taken place. The price, or value, of the comparable business is compared to its earnings, cash flow, or other ratios to derive **market multiple** values for each factor. Those market multiples are then adjusted for the **riskiness** of the subject company in comparison with the comparative company(ies) and multiplied by a **normalized level** of expected earnings and cash flow for the subject company.

The **normalization** of the company's earnings and cash flow is done by analyzing its financial statements over a period of time, taking into consideration fluctuations in the business cycle and adjusting for income or expense items that are extraordinary, nonrecurring, or discretionary. The goal is to use the historical financial results to determine results that would be **representative of the business's future prospects.**

The **riskiness** of the company in relation to the comparable companies is determined by means of both qualitative and quantitative factors. Among these are comparisons of size, leverage, profitability, growth prospects, and quality of management, among others. For example, everything else being equal, a larger company will generally be worth more than a smaller one; and a company with strong growth prospects will generally be worth more than a mature company with not much growth. The qualitative analysis should include information about sales concentration, i.e., whether a few customers account for a large proportion of sales; geographic risk; access to capital, and so forth.

Comparable companies selected should be as similar to the company being valued as possible. Factors to consider are their line of business, geographic location, size, financial similarities and operating similarities. Factors selected as most important will depend upon what type of company it is and what market(s) it serves. For example, if the company being valued is a home builder with a local presence only, it would be very important to find comparable companies located in the same geographic area. However, if the company were a manufacturer with a national market, it would be less important for the comparable companies to be in the same geographical area.

Special attention should be given to the comparisons with companies that are most similar in nature to the subject company. If no one company stands out as more comparable than the others, the analyst should start with the median multiples of all the comparable companies.

There are several variations of the market multiple approach, and each focuses on different earnings or cash flow measures in calculating market multiples. The most commonly used multiples are Price/Earnings and Price/Cash Flow. When P/E or P/CF multiples for the comparable company, adjusted for risk, are multiplied by the subject company's normalized, representative earnings or cash flow, the result should be an indication of the subject company's value.

Other ratios used might be selected in order to evaluate the comparable companies' performance as debt-free companies by eliminating interest expense.

Other approaches such as Price to Book Value, Price to Sales, or Total Invested Capital to Sales ratios can be useful in certain situations, but usually they are not as relevant as multiples that incorporate earnings and cash flow.

Example: Assimilated Stores, Inc. is considering acquiring Takeover Target, Inc. In its analysis of Takeover Target's financial statements, Assimilated has developed the following balance sheet and income statement values as representative values. These representative values are based on normalized historical amounts for the past three years that have been averaged.

BALANCE SHEET

Total Assets	$43,000,000
Total Liabilities	30,000,000
Total Equity	$13,000,000

INCOME STATEMENT

Sales	$70,000,000
Cost of Goods Sold	53,200,000
Gross Profit	$16,800,000
Operating Expenses	8,350,000
EBIT	$ 8,450,000
Interest Expense[k]	950,000
Pre-tax Net Income	$ 7,500,000
Income Tax @ 40%	3,000,000
Net Income	$ 4,500,000

Assimilated has identified five comparable businesses and has calculated Price/Earnings ratios, Price/Book ratios, and Price/Sales ratios for all five. The ranges are as follows:

	Low	High
Price/Earnings	10.0	15.0
Price/Book	3.3	6.5
Price/Sales	.6	.9

After performing a relative risk analysis, comparing Takeover Target's risk factors with the risk factors of the five comparable businesses, Assimilated has determined that Takeover Target is in the higher end of the risk range. Therefore, Assimilated has assigned risk-adjusted ratios at the low end of the range to compensate for the increased risk. The risk-adjusted ratios are multiplied by Takeover Target's earnings, book value and sales to calculate a reasonable price to pay for Takeover Target.

The results are:

Risk-Adjusted Ratio			Multiplied By	=	Indicated Value
Price/Earnings Ratio	=	10.0	Net Earnings of $4,500,000	=	$45,000,000
Price/Book Ratio	=	3.3	Net Book Value of $13,000,000	=	$42,900,000
Price/Sales Ratio	=	0.6	Sales of $70,000,000	=	$42,000,000

Thus, Assimilated calculates that a fair price to pay for Takeover Target should be between $42,000,000 and $45,000,000. If Takeover Target is a publicly-owned company, the price per share would be that total amount divided by the number of shares outstanding.

[k] Interest expense for the representative income statement should be based on only the most current year's interest expense, not an average of 3 years.

The Discounted Cash Flow Approach

The **discounted cash flow** approach, or income approach, involves determining the present value of the future cash flows of the company to be valued. The value of the company is the present value of the expected future cash flows generated by it, discounted at the required rate of return.

In valuing a business, we use **free cash flow,** which is **cash flow before interest but after taxes and after capital expenditures**. It is the cash flow that remains after we subtract from the expected revenues the expected operating costs and the capital expenditures required to sustain the cash flows. Furthermore, it is **cash flow**, not accounting income, that we look at. However, if all we have is Earnings Before Interest and Taxes (EBIT), then we start with that, adjusted for taxes. To convert EBIT to a usable cash flow amount, we adjust for capital expenditures, depreciation, and change in non-cash working capital. The future expected annual capital expenditures amount needs to be reduced by depreciation, and the net amount is a reduction to free cash flow. And if we have a change in the non-cash components of working capital (a decrease to cash if working capital goes up and an increase to cash if working capital goes down), we build that into the calculation, as well.

	EBIT (1 – tax rate)
–	(Capital Expenditures – Depreciation)
+/–	<u>Change in Non-Cash Working Capital</u>
=	Free Cash Flow

Estimates of free cash flows need to include considerations for any synergistic effects if the contemplated combination takes place. For instance, if there will be economies of scale, then some of the expenses can be eliminated. Thus, the expected free cash flows should be **incremental** free cash flows (before interest expense, after taxes, and after capital expenditures) that will result from the proposed combination.

The reason interest expense is not included as a reduction in free cash flow for this purpose is because any liabilities of the acquired company will be included as a reduction of the ultimate valuation, if they are to be assumed by the acquirer. The present value of the future cash flows represents the maximum cash price to be paid for the business, **if** there are no liabilities. If the acquiring firm is assuming liabilities, the liabilities should be adjusted to their market value and subtracted from this cash price. And just as is done in capital budgeting, financing arrangements made for the purchase of the business are not a part of the valuation consideration but should be separate from the calculation of the worth of the investment.

Required Rate of Return

The discount rate to be used in calculating the present value of the future cash flows is known as the **required rate of return.** A firm should invest money in an acquisition only if the acquisition provides a return higher than the required rate of return of stockholders. Thus, the rate should reflect the market's expected rate of return plus a **risk premium** for this particular investment. The rate should be the **cost of equity of the acquired firm** in order to reflect the riskiness of the acquired firm's cash flows.

The cost of equity of the firm to be acquired can be approximated by means of either the **Capital Asset Pricing Model (CAPM)**, or **Arbitrage Pricing Theory (APT)**.

Recall that the **Capital Asset Pricing Model (CAPM)** is a means of determining an individual company's cost of equity capital by adjusting the risk-free rate by a factor representing the additional compensation required for the systematic (undiversifiable) risk of the company's stock. The greater the systematic risk of a security, the greater will be the return that investors will require from the security.

The formula is:

$$\mathbf{R = R_F + \beta(R_M - R_F)}$$

Where:

R	=	Required rate of return on equity capital
R_F	=	Risk-free rate (i.e., government securities)
R_M	=	Market rate
β	=	The *beta* of the firm (a measure of the stock's volatility compared with the volatility of the market)
$R_M - R_F$	=	Risk Premium

> Note: If you are expected to use the Capital Asset Pricing Model to estimate a firm's cost of equity capital in a problem, the *beta* β, the Risk-Free Rate, and the Market Rate will be given in the problem.

Arbitrage Pricing Theory is an alternative method of calculating the cost of equity capital. It is based on the theory that a competitive financial market will assure equilibrium pricing based on risk and return factors, because of arbitrage.

Arbitrage means that if you can find two things that are essentially the same but one is less expensive than the other, you can buy at the cheaper price while simultaneously selling at the more expensive price and pocket the difference.

In Arbitrage Pricing Theory, the price of any asset depends on multiple factors, and arbitrage efficiency controls the price.

Arbitrage Pricing Theory looks at a number of common risk factors to determine the correct price for a security, with the goal of then using that information to identify securities that are underpriced and can be purchased and immediately resold for a higher price. Arbitrage Pricing Theory does not state what these factors are. They could be anything that affects stock price. The return on the market portfolio might be one factor, or it might not be.

> APT says that the risk premium for any investment depends upon its sensitivity to several risk factors representing systematic, or unavoidable, risk, and the expected risk premium for each factor.

Many of the terms in the APT formula operate in ways that are similar to the ways that the terms operate in the Capital Asset Pricing Model. However, we identify them differently simply because they represent a different theory.

The APT formula, if there are two risk factors, is:

$$\mathbf{R = a + b_1r_1 + b_2r_2}$$

Where:

R	=	Expected Rate of return,
a	=	The risk-free rate of return
$b_{1,2}$	=	Coefficients that represent the amount of change in the security's return for a one-unit change in the risk factor
$r_{1,2}$	=	Expected risk premium associated with each systematic risk factor

The expected risk premium associated with each factor is the excess required return above the risk-free rate associated with that risk factor.

The expected risk premium for a given factor might be negative, if the factor decreases risk rather than increasing it. In the event this occurred, the negative risk premium would indicate that **value** was associated with the factor, and its presence would **decrease** the expected return on the security.

If there are more than two factors, the expected return is calculated in the same way, by multiplying each risk premium by its coefficient, summing the results, and adding the result to the risk-free rate (a).

Arbitrage Pricing Theory does not specify what the major factors are that would affect expected return. However, several factors that have been suggested are:

- Yield spread, or the long-term U.S. Treasury Bond rate minus the 30-Day Treasury Bill rate;
- Fluctuations in interest rates;
- Exchange rate fluctuations;
- Changes in forecasts of real GNP; and
- Forecasts of the inflation rate.

Arbitrage Pricing Theory can also be used to calculate the **risk premium only** (excluding the risk-free return part). The rate calculated by this formula is equal to the difference between the expected return to the market and the risk-free rate, and it represents only the systematic risk that is **undiversifiable**. In this case, the formula is:

$$\textbf{Expected risk premium} = \mathbf{r - r_f}$$

$$= b_1(\text{Factor }_1 - r_f) + b_2(\text{Factor }_2 - r_f) + b_3(\text{Factor }_3 - r_f) + \cdots$$

Expected Growth Rate

In valuing a business using the present value of future cash flows, analysts usually split the forecasting into two stages, similar to the way we calculated the two-stage dividend discount model previously.

The first stage includes detailed annual forecasts of financial statement items up to some **horizon date**. The second stage includes forecasts beyond the horizon date to infinity, using a single growth rate forecast for all years beyond the horizon date. Using the two sets of forecasts, then, the present value of all the future cash flows is determined in order to estimate the firm's intrinsic value.

It is relatively simple to determine the present value of the cash flows in the first stage, but for the second stage, we need to determine the value of cash flows that are growing "in perpetuity." We can do that, even though we have no end in sight to the cash flow, if we know the growth rate in the cash flows. Because this determination depends upon an annual growth rate forecast, the **expected growth rate** is one of the most important inputs into the valuation.

The formula to determine the value of the cash flows beyond the horizon date, called the **Gordon Growth Model**, is a slight adaptation of the constant growth dividend discount model.

$$\textbf{Present Value of cash flows growing "in perpetuity"} = \frac{\textbf{Expected Free Cash Flow for the Next Year}}{\textbf{Cost of Capital} - \textbf{Expected Growth Rate}}$$

We will look at each of the components of this formula in turn.

If the expected growth rate is not given, it can be calculated. The expected growth rate is a function of two things:

1) Earnings retained and reinvested; and
2) Return earned on the earnings retained and reinvested.

The expected growth in EBIT will be the Expected Rate of Reinvestment of Earnings × the Expected Rate of Return on Capital Invested.

To calculate the rate of expected growth in EBIT, however, we may first have to calculate the Rate of Reinvestment and/or the Return on Capital, if either or both of those are not given.

Rate of Reinvestment of Earnings

The reinvestment rate may be derived in more than one way. The correct way depends on what information is given.

- If a dividend payout ratio is given, the reinvestment rate is 1 – the dividend payout ratio.
- If a dividend payout ratio is not given but information on capital expenditures, depreciation, and change in noncash working capital is given, then the reinvestment rate is (Capital Expenditures – Depreciation + Increase in Noncash Working Capital OR – Decrease in Noncash Working Capital) / EBIT (1 – tax rate).

$$\textbf{Reinvestment Rate} = \frac{\textbf{Capital Expenditures} - \textbf{Depreciation} \ \textbf{+or–} \ \Delta \ \textbf{Non-Cash WC}}{\textbf{EBIT} \ (1 - \textbf{tax rate})}$$

Return on Capital Invested

Total capital invested is debt plus equity, which is the same as total assets.

$$\textbf{Return on Capital Invested} = \frac{\textbf{EBIT} \ (1 - \textbf{tax rate})}{\textbf{Total Capital Invested}}$$

Growth Rate

Both the Rate of Earnings Reinvestment and the Rate of Return on Capital Invested should be future, not past. Multiplying the two amounts together, we get the expected growth rate.

Note that if you are using the formula above incorporating Capital Expenditures, Depreciation and change in Non-Cash Working Capital to calculate the Reinvestment Rate, when it comes time to multiply the Reinvestment Rate by Return on Capital Invested, the denominator in the Reinvestment Rate formula cancels out the numerator in the Return on Capital Invested formula, leaving the following formula for the Growth Rate:

$$\textbf{Growth Rate} = \frac{\textbf{Capital Expenditures} - \textbf{Depreciation} \ \textbf{+or–} \ \Delta \ \textbf{Non-Cash WC}}{\textbf{Total Capital Invested}}$$

Cost of Capital

The cost of capital is the **required rate of return on equity** for the potential acquisition which, as explained above, can be calculated using either the Capital Asset Pricing Model or Arbitrage Pricing Theory.

We should now be able to return to our formula for the value of a stream of cash flows "in perpetuity," and calculate it using the elements as determined above:

$$\textbf{Present Value of cash flows growing "in perpetuity"} = \frac{\textbf{Expected Free Cash Flow for the Next Year}}{\textbf{Cost of Capital} - \textbf{Expected Growth Rate}}$$

Note that "present value" in the formula above does not mean that the cash flows have been discounted to the present, at least, not yet. Since this is the second stage of the analysis, these cash flows are being discounted to the end of the first stage. After that has been done, stage one and stage two will be discounted together to the present.

Example #1: Assimilated Stores, Inc. is considering acquiring Takeover Target, Inc. Takeover Target has the following estimated incremental future earnings and investments (in millions of dollars):

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6
Assets	75.0	87.6	101.6	117.0	137.3	161.1
EBIT (1−t)	18.0	20.0	22.0	29.0	34.0	38.0
Less: Investments (Cap Exp − Depr)	12.6	14.0	15.4	20.3	23.8	26.6
Free Cash Flow	5.4	6.0	6.6	8.7	10.2	11.4

Seventy percent of Takeover Target's earnings will be reinvested each year beyond Year 6. It is expected that Earnings Before Interest and Taxes Net of Tax will be 15% of assets each year beyond Year 6. If Takeover Target's cost of equity capital is 15%, what is the value of the business?

Solution:

1. We discount the expected Free Cash Flows for Years 1 through 5 back to the present (Year 0) using Takeover Target's cost of equity capital of 15%.

$$\frac{5.4}{1.15} + \frac{6.0}{1.15^2} + \frac{6.6}{1.15^3} + \frac{8.7}{1.15^4} + \frac{10.2}{1.15^5} =$$

$$4.35 + 4.54 + 4.34 + 4.97 + 5.07 = \mathbf{\$23.27\ million}$$

2. We determine the growth rate in earnings for each year beyond Year 6 by multiplying the 70% reinvestment rate by the 15% return on assets. .70 × .15 = .105. The growth rate is 10.5%.

3. We now pretend that Year 6 is Year 1 and the end of Year 5 is Year 0. We calculate the value of the Free Cash Flow for Years 6 and beyond as of the end of Year 5, using the Gordon Growth Model, the 15% required rate of return, and the growth rate of 10.5%. When we did this step for the dividends, we estimated the numerator of the calculation by increasing the last projected dividend amount we had by the growth rate. Here, though, we have a projection of free cash flow for Year 6, so that becomes our "next year's" free cash flow amount for this calculation:

$$\text{Value at end of Year 5} = \frac{11.4}{.15 - .105} = \$253.3\text{ million}$$

3. We discount this $253.3 million value at the end of Year 5 back to the present, to Year 0:

$$\frac{253.3}{1.15^5} = \mathbf{\$125.93\ million}$$

4. Finally, we add together the $23.27 million calculated in Step 1 and the $125.93 million calculated in Step 2 to calculate the value of the business:

$23.27 million + $125.93 million = $149.2 million

If the acquired company has liabilities that are being assumed by the acquiring company, those liabilities should be adjusted to market value and **subtracted from the present value of the free cash flows** to arrive at the maximum price that should be paid for the business. Any price up to this net amount should result in a worthwhile investment.

Addressing Risk in Business Valuation

What if the revenues and earnings of the business turn out to be not as predictable as we thought? What if the growth rate turns out to be lower than 10.5%, or if the free cash flow projections turn out to be too high? That would clearly impact the valuation calculated using discounted cash flow methods as above. We could easily end up paying too much for Takeover Target, even though we did everything right in our calculations.

In the capital budgeting section of this textbook, we said the following:

"Companies adjust for risk by using **risk-adjusted discount rates.** A company will **increase the discount rate** used in NPV calculations for more risky, or uncertain, investments. A higher discount rate will require higher expected future cash flows for the company to make the investment, thus making fewer investments acceptable. And it will lower the discount rate used for an investment that is judged less risky than the company's present portfolio of investments."

We do the same thing when calculating the value of a business we are considering purchasing. If the projected cash flow of the business and/or other variables used in the valuation analysis are not very predictable, we need to assign a **risk premium** to the cost of capital/required rate of return used to discount the future expected cash flows. By increasing the discount rate, we will decrease the resulting valuation and may bring the offering price down to what we consider to be a more reasonable level.

Here is the discounted cash flow analysis again, this time calculated using a 25% cost of capital as the discount rate:

Example #2: Assimilated Stores, Inc. is considering acquiring Takeover Target, Inc. Takeover Target has the following estimated incremental future earnings and investments (in $millions):

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5	Yr 6
Assets	75.0	87.6	101.6	117.0	137.3	161.1
EBIT (1−t)	18.0	20.0	22.0	29.0	34.0	38.0
Less: Investments (Cap Exp − Depr)	12.6	14.0	15.4	20.3	23.8	26.6
Free Cash Flow	5.4	6.0	6.6	8.7	10.2	11.4

Seventy percent of Takeover Target's earnings will be reinvested each year beyond Year 6. It is expected that Earnings Before Interest and Taxes Net of Tax will be 15% of assets each year beyond Year 6. If Takeover Target's cost of equity capital of 15% plus a 10% risk premium is **25%**, what is the value of the business?

Solution:

1. We discount the expected Free Cash Flows for Years 1 through 5 back to the present (Year 0) using Takeover Target's cost of equity capital of **25%**.

$$\frac{5.4}{1.25} + \frac{6.0}{1.25^2} + \frac{6.6}{1.25^3} + \frac{8.7}{1.25^4} + \frac{10.2}{1.25^5} =$$

$$4.32 + 3.84 + 3.38 + 3.56 + 3.34 = \mathbf{\$18.44\ million}$$

2. We determine the growth rate in earnings for each year beyond Year 6 by multiplying the 70% reinvestment rate by the 15% return on assets. .70 × .15 = .105. The growth rate is 10.5%.

3. We now pretend that Year 6 is Year 1 and the end of Year 5 is Year 0. We calculate the value of the Free Cash Flow for Years 6 and beyond as of the end of Year 5, using the Gordon Growth Model, the 25% required rate of return, and the growth rate of 10.5%. When we did this step for the dividends, we estimated the numerator of the calculation by increasing the last projected dividend amount we had by the growth rate. Here, though, we have a projection of free cash flow for Year 6, so that becomes our "next year's" free cash flow amount for this calculation:

$$\text{Value at end of Year 5} = \frac{11.4}{.25 - .105} = \$78.6 \text{ million}$$

3. We discount this $78.6 million value at the end of Year 5 back to the present, to Year 0:

$$\frac{78.6}{1.25^5} = \mathbf{\$25.76\ million}$$

4. Finally, we add together the $18.44 million calculated in Step 1 and the $25.76 million calculated in Step 2 to calculate the value of the business:

$18.44 million + $25.76 million = <u>$44.2 million</u>

Because we used 25% instead of 15% as the discount rate to discount the cash flows in Example #2, the value of the business that resulted is $105 million **lower** than it was in Example #1.

If we had considered that there was very little risk of the forecasted variables being different from our forecast, we might have tried **reducing** the discount rate to 10% instead of increasing it to 25%. Reducing the discount rate to 10% would have resulted in a **higher** valuation for the business.

When using discounted future expected cash flows to determine the value of a business or of a stock, the discount rate used can have a very pronounced effect on the valuation that results.

- A **higher** discount rate results in a **lower** present value.
- A **lower** discount rate results in a **higher** present value.

Sensitivity Analysis in Business Valuation

Sensitivity analysis can be used to test the assumptions used in the valuation model. As we have said previously, sensitivity analysis is a "what if" technique. The key assumptions, such as the 10.5% growth rate, the $11.4 million free cash flow projection for Year 6 or any of the other free cash flow projections for the other years, or the discount rate used, can be changed, leaving the other assumptions unchanged.

After changing one assumption, we recalculate the valuation with the changed assumption to determine what effect the change has on the value.

Example #2 above is a sensitivity analysis that shows what happens to the value of Takeover Target when we change the discount rate from 15% to 25%, leaving all other assumptions the same.

Other Considerations in Valuing a Potential Acquisition

Income Taxes

Businesses that have reported losses on previous years' tax returns have what is called **tax loss carryforwards** that they can benefit from[1]. In the U.S., taxes are paid on taxable income. If taxable income is zero or less than zero, no tax is due. However, taxable income of less than zero (a loss) can be used on a future tax return to offset future income, thereby decreasing future taxable income. A decrease in future taxable income means a decrease in future income tax due, and that means increased future cash flows. Therefore, a business with past tax losses may have an increased value as an acquisition, because future expected cash flows from the business will be higher than they would have been without the past tax losses.

Of course, the acquiring firm must have enough taxable income to be able to make use of the tax loss carryforwards. If the acquiring firm's taxable income is not high enough to be able to use them, the tax loss carryforwards from the acquisition will be useless and therefore valueless and thus, this premium should not be paid for the purchase of the target firm.

Tax benefits can also result from the fact that as a result of a merger, the assets of the acquired firm are written up to their market values and depreciated on the basis of their new values, resulting in larger depreciation writeoffs and thus lower taxable income and lower taxes going forward.

Real Options

Just as the existence of a real option can increase the net present value of a capital project, the existence of a real option can increase the value of a potential acquisition. Examples of real options that can increase the value of a potential acquisition are:

- A patent or patents owned by the target firm that could be licensed, sold or put into production;
- Natural resource reserves that may be available to the target firm that could be developed;
- An expansion option that the potential acquisition may have as a result of a limited new product introduction presently in progress; or
- An option to abandon a project currently underway, if the cash flows do not meet expectations;

Essentially any real option available to the potential acquisition would accrue to the acquiring firm. The value of real options should be included in the valuation of the firm.

Summary of the Discounted Cash Flow Approach to Business Valuation

The discounted cash flow approach is the most theoretically sound approach to valuing a business. A drawback to this approach, though, can be difficulty in determining accurate future expected returns and the required rate of return. However, it is the best approach to use in two circumstances: (1) when revenues and earnings of the business to be valued are highly predictable, making estimation of expected future cash flows relatively easy; or (2) when no comparative sales of businesses can be found.

When revenues and earnings of the business are not highly predictable, the discounted cash flow approach can still be used, but a **risk premium** needs to be added to the required rate of return used to discount the expected future cash flows.

[1] At least under present tax law, businesses can benefit from tax loss carryforwards. The tax law could be changed in the future.